

IMPROVEMENT OF PERFORMANCES OF MICROSTRIP STRUCTURES  
BY EQUALIZATION OF PHASE VELOCITIES.

by A.E. ROS and D. POMPEI  
Laboratoire d'Electronique  
Faculté des Sciences et des Techniques  
Parc Valrose  
06034 NICE CEDEX - France

ABSTRACT.

It is well known that, in the TEM approximation, the electrical characteristics of microstrips couplers are easily obtained, if we assume the equality between the odd and even modes phase velocities. In the other hand the directivity of such couplers is considerably decreased when the gap between these velocities increased. So, it can be very useful to equalize them. This result can be achieve by using a dielectric overlay or an upper ground plane symmetrical of the lower one with regard to the conductors. This last result can be generalized to a n-parallel microstrip device. Advantages and disadvantages of these two technics are discussed and experimental results are given.

Introduction

For the analysis or synthesis of microstrip couplers in the case of the TEM approximation, it is generally considered that the phase velocities of the two modes are equal such as the formulas giving the matching impedance and the coupling coefficient are simple. These values can be expressed in terms of the characteristics impedance  $Z_{oe}$  and  $Z_{oo}$  and the wave numbers  $\beta_{oe}$  and  $\beta_{oo}$  of the two modes.

The classical approach for the description of a microstrip coupler leads to the definition of the scattering matrix S. This matrix connect the reflected voltages and currents to the incident ones and for the reflected to incident voltages, if  $s_{ij}$  is the ratio of the reflected voltage at the port i to the incident one at the port j, we have :

$$A_{11} = \frac{\rho_e}{2} \left[ 1 - \frac{1 - \rho_e^2}{\exp(2j\beta_{oe}l) - \rho_e^2} \right] + \frac{\rho_o}{2} \left[ 1 - \frac{1 - \rho_o^2}{\exp(2j\beta_{oo}l) - \rho_o^2} \right]$$

$$A_{21} = \frac{(1 - \rho_e^2) \exp(j\beta_{oe}l)}{2[\exp(2j\beta_{oe}l) - \rho_e^2]} + \frac{(1 - \rho_o^2) \exp(j\beta_{oo}l)}{2[\exp(2j\beta_{oo}l) - \rho_o^2]}$$

$$A_{31} = \frac{\rho_e}{2} \left[ 1 - \frac{1 - \rho_e^2}{\exp(2j\beta_{oe}l) - \rho_e^2} \right] - \frac{\rho_o}{2} \left[ 1 - \frac{1 - \rho_o^2}{\exp(2j\beta_{oo}l) - \rho_o^2} \right]$$

$$A_{41} = \frac{(1 - \rho_e^2) \exp(j\beta_{oe}l)}{2[\exp(2j\beta_{oe}l) - \rho_e^2]} - \frac{(1 - \rho_o^2) \exp(j\beta_{oo}l)}{2[\exp(2j\beta_{oo}l) - \rho_o^2]}$$

$$\text{where } \rho_e = \frac{Z_{oe} - Z_o}{Z_{oe} + Z_o} \quad \rho_o = \frac{Z_{oo} - Z_o}{Z_{oo} + Z_o}$$

The matching condition is obtain for  $s_{11} = 0$  and the matching impedance  $Z_o$  is then given by :

$$\frac{\rho_e \exp(2j\beta_{oe}l) - 1}{2 \exp(2j\beta_{oe}l) - \rho_e^2} = - \frac{\rho_o \exp(2j\beta_{oo}l) - 1}{2 \exp(2j\beta_{oo}l) - \rho_o^2}$$

The coupling coefficient in dB to port 3 is :

$$K = 20 \log_{10} |s_{31}|$$

and the directivity in dB between port 3 and 4 is

$$D = 20 \log \left| \frac{s_{31}}{s_{41}} \right|$$

These expressions are really complicated so, it is generally considered that the coupler is ideal that is to say that the wave number (or phase velocity :  $\beta = \frac{\omega}{V}$ ) is the same for the even and odd modes. The length of

the coupling region at the central frequency is equal to  $\frac{\lambda_{ge}}{4} = \frac{\lambda_{go}}{4}$ , so  $\beta_{oe}^{-1} = \beta_{oo}^{-1} = \frac{\pi}{2}$

The matching condition becomes :

$$\frac{\rho_e^2}{1 + \rho_e^2} = - \frac{\rho_o^2}{1 + \rho_o^2} \quad \text{or} \quad \rho_e = - \rho_o$$

and we obtain the classical matching condition :

$$\rho_e^2 = Z_{oe} Z_{oo}$$

The coupling coefficient has also a simple expression :

$$K = 20 \log_{10} \left| \frac{2 \rho_e}{1 + \rho_e^2} \right| = 20 \log_{10} \left| \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} \right|$$

The directivity then infinite, because :

$$s_{41} = \frac{j}{2} \left[ \frac{\rho_e^2 - 1}{\rho_e^2 + 1} - \frac{\rho_o^2 - 1}{\rho_o^2 + 1} \right] = 0 \quad \text{if} \quad \rho_e = - \rho_o$$

Phase velocities in parallel microstrips devices.

Indeed, the velocities of the two modes are generally different and the above expressions becomes inapplicable. In particular, the directivity rapidly decreased when the gap between the velocities increases. In the case of the coupler, this gap can reach 9 % for the small values of  $\epsilon_r$  (strong coupling) and high dielectric permittivity of the substrate (Table I).

TABLE I

W/H	S/H	phase velocities in m/s ( $\times 10^{-8}$ )		$\frac{\Delta v}{v}$	$\epsilon_r$
		odd mode	even mode		
0.5	0.25	2.2101	2.1276	4%	$\epsilon_r = 2.65$
	1	2.1948	2.0896	5%	
	0.5	2.1849	2.0866	5%	
	2	2.1668	2.0674	5%	
0.5	0.25	1.2873	1.2048	7%	$\epsilon_r = 9.7$
	1	1.2702	1.1682	9%	
	0.5	1.2602	1.1655	8%	
	2	1.2420	1.1480	8%	

For multiconductors microstrip devices, the gap between the higher and the lower phase velocities is also no negligible for strong coupling and high dielectric constant (Table II and III).

TABLE II

W/H	S/H	phase velocities in m/s ( $\times 10^{-8}$ )			$\frac{v_1 - v_3}{v_2}$	$\epsilon_r$
		$v_1$	$v_2$	$v_3$		
0.5	0.1	2.1904	2.1831	2.1252	3 %	$\epsilon_r = 2.65$
	0.2	2.1847	2.1504	2.0306	7 %	
	0.4	2.1609	2.0954	1.9611	9 %	
0.5	0.1	1.3027	1.2953	1.2380	5 %	$\epsilon_r = 9.7$
	0.2	1.2969	1.2626	1.1505	11 %	
	0.4	1.2730	1.2106	1.0916	15 %	

TABLE III

Phase velocities in m/s ( $\times 10^{-8}$ ) and gap in %.				
	$\epsilon_r = 2.65$	$\epsilon_r = 9.7$		
	$v_i$	$\frac{v_i - v_5}{v_3}$	$v_i$	$\frac{v_i - v_5}{v_3}$
W/H = 1	2.1872		1.2993	
S/H = 0.4	2.1784		1.2905	
	2.1530	9 %	1.2652	14 %
	2.0920		1.2072	
	1.9938		1.1190	

For these cases, the separation between two successive lines are the same and the lines have the same widths.

In order to simulate an ideal microstrip coupler or a coupler in an homogeneous medium and to obtain a broadband coupler <sup>3,4,5</sup>, it should be very interesting to equalize the phase velocities of the modes.

The gap between these velocities is caused by the dissymmetry of the structure. There is a concentration of the electric lines in the substrate so that the propagation is slowed down by comparison with the case without dielectric but with different rate according to we are considering the odd or even mode.

The problem is to obtain a configuration as close as possible to an homogeneous configuration. So one solution is to use a dielectric overlay of high permittivity such as the effective dielectric constant of the upper space tends to the permittivity of the substrate under the lines. Another solution will be to 'symmetrize' the configuration such as the gap between the lines and the upper ground plane has the same height than the dielectric substrate.

#### Dielectric overlays.

In this case, we use a thin layer of high permittivity above the conductors. We have to equalize the effective dielectric constant of the upper and the lower space on each side of the microstrip lines. The height  $H$  and the permittivity  $\epsilon_r$  of the substrate in the lower space are fixed, so we have to vary the height  $H_1$  or permittivity  $\epsilon_r$  of the dielectric overlay in order to simulate an homogeneous medium and to equalize the phase velocities of the two modes.

#### Symmetrical configurations.

In 1975, Arain and Spencer <sup>6</sup> reported that it is possible to improve the performances of a microstrip coupler if the upper and lower ground plane are symmetrical with respect to the lines. In fact, in the same time we have been able to verify that in this case the phase velocities of the two modes are equal (Fig. 2,3). If  $H_1$  is the height of the air gap above the conductors, when  $H_1 = H$  the effective dielectric constant is equal to  $\epsilon_{eff} = \frac{\epsilon_r + 1}{2}$  and then the phase velocities should

be equal to  $v_e = v_o = \sqrt{\frac{c}{\epsilon_{eff}}} = c \sqrt{\frac{2}{\epsilon_r + 1}}$

so if  $\epsilon_r = 15$   $\epsilon_{eff} = 8$   
and  $v_e = v_o = 1.0599$  m/s.

The crossing point on the figure 4 gives for the common value of  $v$  and  $v_e$  : 1.06 m/s.

This very good agreement has been obtained for any configuration we have tested.

In fact, this result can be achieved by considering the expression of the phase velocity in terms of the capacities of the line in the homogeneous case (without dielectric) :  $C_0$  and in the inhomogeneous case :  $C$ .

$$v_p = c \sqrt{\frac{C_0}{C}}$$

If  $\Gamma_{11}$  is the capacitance of the line in front of the ground plane and  $\Gamma_{12}$  is the mutual capacitance between the two lines. The even and odd modes capacitances can be written :

$$C_e = \Gamma_{11} \text{ and } C_o = \Gamma_{11} + 2 \Gamma_{12}$$

When  $H_1 = H$ ,  $\Gamma_{11}$  can be split into three capacitances :  $\Gamma_{11}$  for the capacitance between the strip and the upper ground plane,  $\Gamma_{11}$  between the strip and the lower ground plane and  $C_b$  taking into account the edge effect. We have yet shown that  $\Gamma_{11}$  (and  $\Gamma_{12}$ ), and consequently  $C_b$  are linear function of  $\epsilon_r$  <sup>7,8</sup>, so  $C_b$  can be included in  $\Gamma_{11}$  and  $\Gamma_{11}$  such as

$$\Gamma_{11} = \Gamma'_{11} + \Gamma''_{11}$$

In the presence of the dielectric substrate,  $\Gamma'$  stays unchanged,  $\Gamma''_{11}$  becomes  $\epsilon_r \Gamma''_{11}$  and  $\Gamma_{12}$  becomes  $\frac{\epsilon_r + 1}{2} \Gamma_{12}$  by a reason of symmetry.

$$\text{So we have } v_e = c \sqrt{\frac{\Gamma'_{11} + \Gamma''_{11}}{\Gamma'_{11} + \epsilon_r \Gamma''_{11}}}$$

$$v_o = c \sqrt{\frac{\Gamma'_{11} + \Gamma''_{11} + 2 \Gamma_{12}}{\Gamma'_{11} + \epsilon_r \Gamma''_{11} + 2 \cdot \frac{\epsilon_r + 1}{2} \Gamma_{12}}}$$

$$\text{when } H_1 = H ; \quad \Gamma'_{11} = \Gamma''_{11}$$

$$\text{and } v_e = c \sqrt{\frac{2 \Gamma'_{11}}{\Gamma'_{11} (1 + \epsilon_r)}} = c \sqrt{\frac{2}{\epsilon_r + 1}}$$

$$v_o = c \sqrt{\frac{2 \Gamma'_{11} + 2 \Gamma_{12}}{(1 + \epsilon_r) \Gamma'_{11} + (1 + \epsilon_r) \Gamma_{12}}} = c \sqrt{\frac{2}{\epsilon_r + 1}}$$

#### Generalization to n-parallel microstrip lines.

We have shown <sup>9</sup> that the electromagnetic properties of n-parallel microstrip lines structures, in the quasi TEM application, can be obtained from the capacitance matrix  $[C]$ . This matrix allows to calculate the inductance matrix  $[M]$  by  $[M] = \frac{1}{c^2} [C_o]^{-1}$  where  $[C_o]$  is the matrix  $[C]$  for the vacuum. From the diagonalization of the product  $[C][M] = \frac{1}{c^2} [C][C_o]^{-1}$  we can reach the n-phase velocities.

The studies, developed in our laboratory have shown that the elements  $C_{ij}$  of the matrix  $[C]$  are linear functions of the dielectric constant of the substrate  $\epsilon_r$ . This result has been obtained whatever the chosen configurations and for  $\epsilon_r$  as large as 70 and up to  $n = 8$ .

So we can write :

$$C = (\epsilon_{r-1}) [A] + [C_o]$$

where  $[A]$  is a  $n \times n$  matrix depending only on the geometrical configuration. The study of the elements  $A_{ij}$  of  $[A]$  has been performed and we have studied their variation in terms of the ratio  $H_1/H$ . After about hundred tests, we have established that for  $H_1 = H$ ,  $A_{ij} = 1/2 C_{ij}$  whatever  $i$  and  $j$  are. (Fig. 4). This result is very important because it allows to show that for  $H_1 = H$ , all the phase velocities of the propagation modes are equal :

$$[G] = [C][M] = \frac{1}{c^2} \left\{ \epsilon_{r-1} [A] + [C_o] \right\} \cdot [C_o]^{-1}$$

for  $H_1 = H$ ,  $[A] = \frac{1}{2} [C_o]$ , then :

$$[G] = \frac{1}{c^2} \frac{\epsilon_{r-1}}{2} [C_o] [C_o]^{-1} = \frac{1}{c^2} \frac{\epsilon_{r-1}}{2} [U]$$

But, we have shown <sup>9</sup> that the phase velocities  $v_i$  are :  $v_i^2 = (g_i)^{-2}$ , where  $g_i$  are the eigen values of  $[G]$ .

Finally, for  $H_1 = H$ , we have

$$v^i = c \sqrt{\frac{2}{\epsilon_{r+1}}}$$

#### Experimental results.

The first experiments have been performed with the Dr F.C. De RONDE at the Laboratoire d'Electronique et de Physique Appliquée at Limeil-Brévannes, France. The above theory have been confirmed if the thickness of the microstrip lines is negligible. In fact, if it is not the case, for small ratio  $s/h$ , that is to say for strong coupling (i.e. large gap between the phase velocities), it is necessary to take into account an edge-to-edge capacitance even for the even mode. The corresponding electric field lines are completely in the air, so there is no more symmetry in the distribution of these electric lines between the two regions, under and above the air-dielectric interface. This dissymmetry is generally small and its effect can generally be neglected. The equalization of the phase velocities for  $H_1 = H$  is then obtained in first approximation. Another difficulty is that some parasitic resonance modes can be observed when the upper ground plane is too close to the microstrip lines.

#### Conclusion.

It is now possible to equalize the phase velocities by two methods. One of them use thin layers of high permittivity dielectric which are deposited on the conductors. This method seems to be simple but the thickness of this overlay depends on the characteristics of the lines, so this is not applicable for the non uniform couplers ( $w$  and  $s$  vary along the propagation direction). In the other hand there is no more direct access to the lines for the integration of active components for example.

The second technics use an upper ground plane such as  $H_1 = H$ . By a convenient realization of the device containing the substrate and the lines, this result can easily be obtained ; and this, whatever the printed lines on the substrate are. The direct access to the lines is also maintained.

Furthermore, it seems that the characteristics impedance  $Z$  has a very fast variation around the configuration where  $H_1 = H$ . This effect has not been tested in the case of dielectric overlays.

Nevertheless this second method is probably the best one to equalize the phase velocities because it is easier to realize and more general.

The directivity and the bandwidth of a coupler are so increased and the performances of the device are improved.

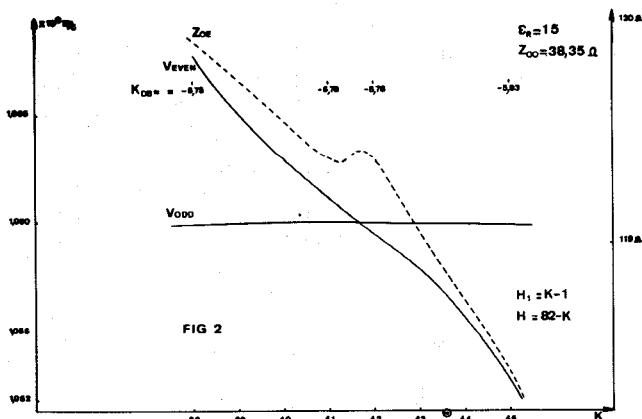


FIG 2

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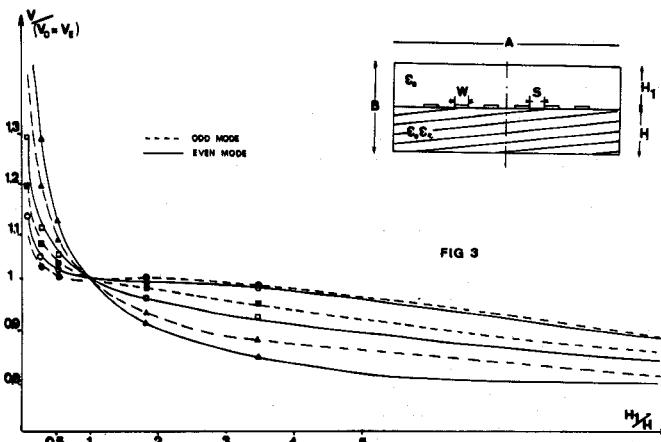


FIG 3

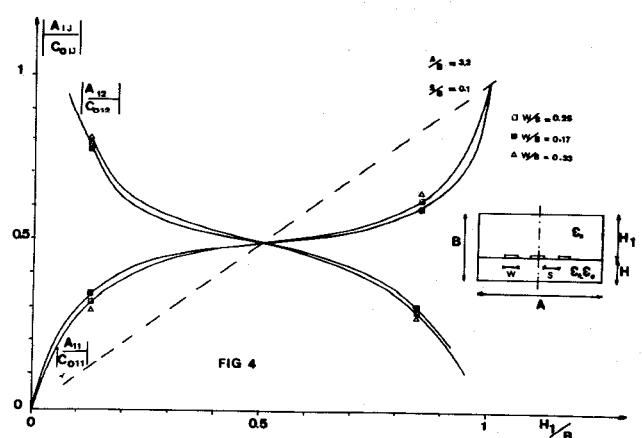


FIG 4